

# MODELING OF SHELL AND TUBE HEAT EXCHANGER USING SYSTEM IDENTIFICATION TECHNIQUES

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**Abstract:** Shell and Tube Heat Exchanger (STHX) is widely used in process industries to withstand higher pressures. Modeling the STHX system is a one of the important task in process control industries and the FOPDT model for the same is obtained by First principles. The development of the model is necessary to study the behavior of the system and to design the suitable controller. The First principles model is developed using the Energy Balance equation of the STHX system and the data collected from the model is used to identify the process. In this work, the cold water flow rate is taken as manipulated variable whereas the hot water outlet temperature is considered as the controlled variable. The aim of this paper is to identify the model of the system based on the observed input-output data. The Nonlinear ARX model, Box-Jenkins model and Output-Error model are obtained from the collected data. Box-Jenkins and Output-Error model originates from linear model identification techniques and Nonlinear ARX model is from nonlinear model identification techniques. The above said models are validated and compared on the basis of Percentage of Fitness, Final Prediction Error (FPE) and Cost Function.

**Keywords:** STHX, FOPDT, Nonlinear ARX, Box-Jenkins, Output-Error, Percentage of Fitness, FPE, Cost Function.

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## 1. INTRODUCTION

STHX is a highly nonlinear process and hence it is always tough to model and control. There are several papers in literature which explains about the modeling and identification techniques. System identification is a method for developing process models, which takes several forms depending upon their dynamics. An attempt was made to identify the difficulty of nonlinear identification. For a good linear model, the linear toolbox guides the user to find a reasonable model [1]. During nonlinear output error model estimation, a modification to the conventional algorithm is proposed in [2] to stabilize the process. Based on the performance analysis (MSE, FPE, Correlation analysis), Nonlinear Auto Regressive eXogenous (NARX) model produces better results for the MIMO ethane ethylene distillation column [3]. The article [4] uses artificial neural network model to predict the heat exchanger process from which the obtained model was validated using performance functions like MSE, SSE (Sum of Squared Error) and the cost function.

To ensure the capability and performance of the heat exchanger, system identification of the process is validated by Hammerstein-Wiener model [5]. System identification methods are used in the design of controllers for regulating the process. The aim of [6] is to identify a model that provides an satisfactory estimation, in the area of application of where it is used. The output-error estimator has the advantage over the prediction-error estimator of being more easily computable. However, the output-error estimator can never be more efficient than the prediction error estimator. The results of paper [7] includes the essential conditions for the output-error estimator and the prediction-error estimator to have the same efficiency, irrespective of the spectral density of the noise process. Recursive Least Squares and Least Squares based Iterative algorithms are used for estimating the parameters of Box-Jenkins models which is clearly explained in [8].

Local linear ARX model identification is done by selecting the input and output data around the selected level. A nonlinear ARX model was identified with its parameters, by integrating with the local linear ARX models. The

dependence of parameters on the input and output is identified numerically and expressed approximately by the polynomials [9]. Input-output data is obtained from the PRBS experiment for the identification of the Shell and tube heat exchanger system using ARMAX model [10]. A mathematical model was selected for industrial heating system by implementing the system identification techniques such as ARX, ARMAX and BJ models. Simulated results proved that the BJ model provides best model in terms of FPE, loss function, percentage of fitness, and co-relation analysis [11]. In [12], Artificial Neural Network (ANN) based approach was applied to estimate the parameters of the pH process. Once the deviations in parameters are frequently identified, Genetic Algorithm (GA) optimally tunes the controller.

This paper is structured in a way over the ideas gained from various system identification techniques through the literature. Modeling the STHX process is shown, from which the obtained model is validated using three System Identification techniques such as Output- Error, Box-Jenkins and Nonlinear ARX model which is explained in the forthcoming sections of this script.

## 2. PIPING AND INSTRUMENTATION DIAGRAM OF STHX

The Shell and Tube Heat exchanger consists of 37 copper tubes and the length of the tubes is of 750 mm with a single pass arrangement. The hot and cold water can be arranged in co-current and counter-current manner. Water is heated to a specific operating temperature in the process tank. The disturbance tank is used to study for disturbance rejection.

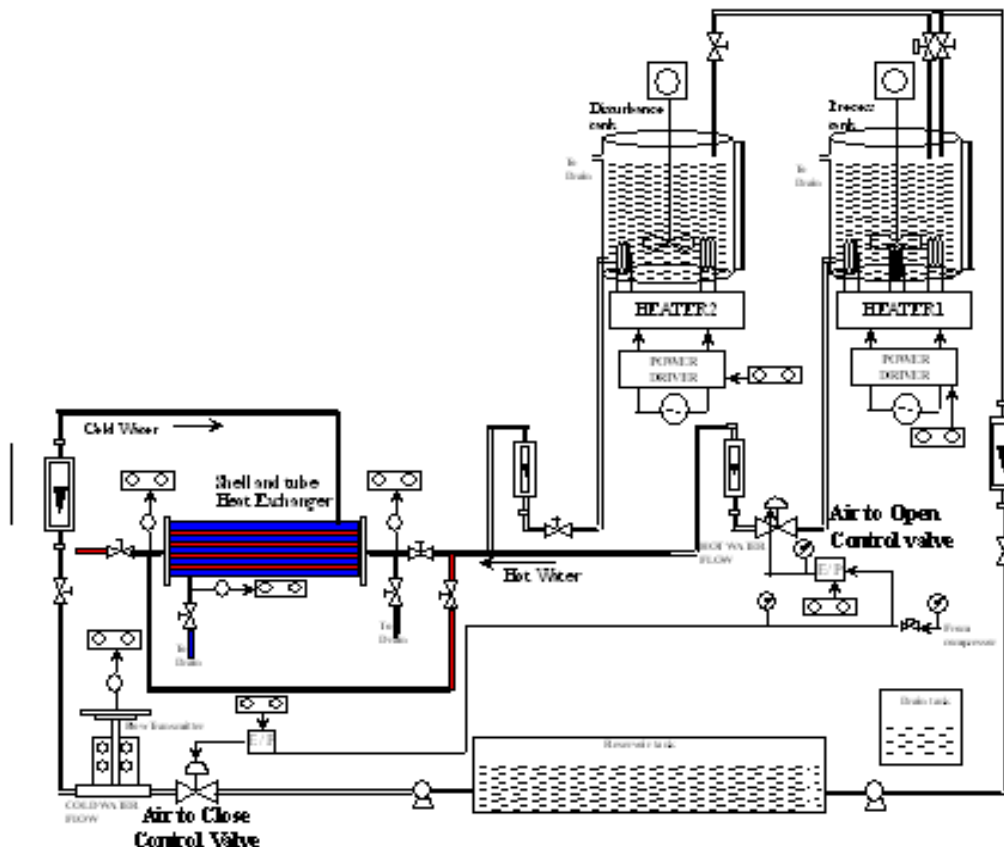


Fig 1: P & ID of STHX

The hot water runs from the process tank and passes through the tube side of the heat exchanger. Cold water is supplied at room temperature and runs from the reservoir tank into the shell side of the heat exchanger. The two power drivers regulate the voltage and current to the heaters, which in turn regulates the temperature of the water in the process and disturbance tank. The inlet and outlet temperatures of the hot and cold water are measured using the Resistance Temperature Detectors. The cold water flow rates are measured using Differential Pressure Transmitter. The cold and hot water inlet flow of the shell side and tube side fluids are manipulated using pneumatic control valves. The controlled variable is hot water outlet temperature whereas the manipulated variable considered here is the cold water flow rate. The process parameters are acquired by means of MATLAB software.

### 3. MATHEMATICAL MODELING

Modeling of a system is necessary for a successful design of a control system. Two types of modeling are available in literature.

#### 3.1 White box modeling

If sufficient knowledge of a system is available, white box model can be developed. This is the mathematical model and it makes use of the first principles (physical laws). This involves the relation between all components of a system.

#### 3.2 Black box modeling

If no information about the system is available, then it is advisable to go for black box model. Here the model is obtained from the behavior of the system (from the input-output data). The output of the system to be modeled is obtained for a known input and from the relationship the model is identified. The process of finding mathematical model for the dynamic system is termed as system identification. It comes under black box modeling. System identification involves some simple mathematical relationship to find out the model of the system from the input-output data after pre-processing of the acquired data. The black box model of the system is determined by the data obtained from the experiment and the knowledge about the system. Identification procedure includes a proper input to the system and after conducting an experiment, the response of the system is to be measured.

In this work, the mathematical model of the STHX is obtained from the energy balance equation. The model is created in MATLAB/Simulink environment. The PRBS signal is applied to the differential equation model developed in MATLAB/Simulink environment and the response is collected. The main aim of this work is to obtain the model of STHX by the system identification procedure from the input-output data. Next step is to select the model and this data is fitted to the selected model. In this case

- (i) Nonlinear ARX model
- (ii) Box-Jenkins model
- (iii) Output-Error model are considered to validate performance of the acquired model. Fitness in terms of %, FPE and cost function for the same is also estimated.

#### 3.3 Energy Balance Equations for STHX system

The STHX consists of two sections namely the shell and tubes. For designing the mathematical model of STHX, the following was assumed. The two sections are separated into small control volumes. These control volumes were assumed to have a constant temperature over that particular volume. Since the STHX is insulated, there is no heat loss from the heat exchanger to the surrounding. Rate of energy stored in the control volume is equal to the rate of gain of energy from the neighboring control volume. To observe the model of the method, hot water tubes are kept at a particular temperature. On that point a little step change in cold water inflow rate is set, both in positive and negative ways to acquire separate response curves.

The Energy Balance Equations for shell-side and tube-side respectively are given below with parameter specifications at the nominal operating point is listed in table 1.

##### 3.3.1 Shell side

$$\frac{\rho_s C_s V_s}{N} * \frac{dT_{co}}{dt} = \dot{m}_s C_s (T_{ci} - T_{co}) + \frac{h_s A_s}{N} (T_{ho} - T_{co}) \quad (1)$$

##### 3.3.2 Tube Side

$$\frac{\rho_t C_t V_t}{N} * \frac{dT_{ho}}{dt} = \dot{m}_t C_t (T_{hi} - T_{ho}) + \frac{h_t A_t}{N} (T_{co} - T_{ho}) \quad (2)$$

#### 3.4 Parameter Specifications

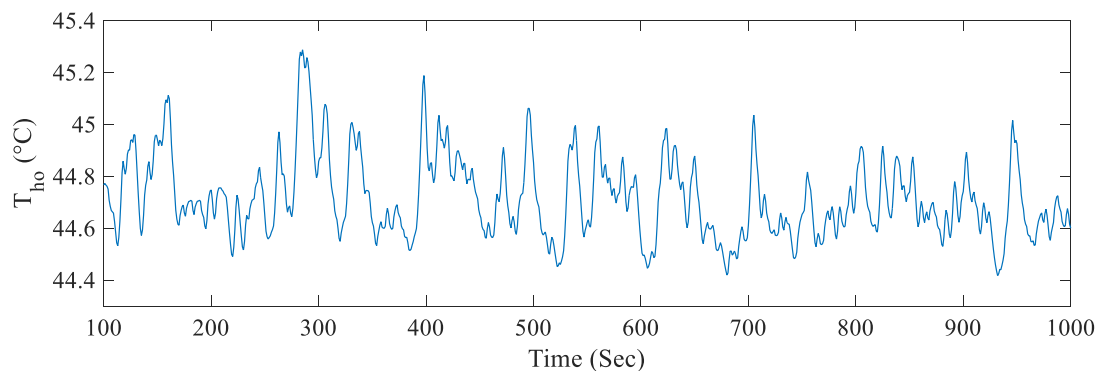
**Table 1: Catalogue data of STHX**

Inputs	Value	Units
Density of Water ( $\rho_s, \rho_t$ )	1000	Kg/m <sup>3</sup>
Specific Heat Capacity of water ( $C_s, C_t$ )	4230	J/Kg °C

Shell Heat Transfer Area ( $A_s$ )	0.281	$m^2$
Tube Heat Transfer Area ( $A_t$ )	0.253	$m^2$
Shell side volume ( $V_s$ )	$2.62 \cdot 10^{-4}$	$m^3$
Tube side volume ( $V_t$ )	$1.43 \cdot 10^{-4}$	$m^3$
Heat transfer coefficient of shell ( $h_s$ )	2162	$W/m^2 \cdot ^\circ C$
Heat transfer coefficient of tube ( $h_t$ )	2162	$W/m^2 \cdot ^\circ C$
Mass flow rate of cold water ( $\dot{m}_s$ )	0-0.12	Kg/S
Mass flow rate of hot water ( $\dot{m}_t$ )	0.0282	Kg/S
Cold water inlet temperature ( $T_{ci}$ )	33	$^\circ C$
Hot water inlet temperature ( $T_{hi}$ )	55	$^\circ C$
Number of control volume (N)	10	NA

#### 4. PRBS EXPERIMENT

Cold water flow rate is given as the input in the form of Pseudo Random Binary Sequence (PRBS) signal and the response of the process to the change in flow rate is acquired. The specifications of the parameters are listed in Table 1. The response of the system for the PRBS input is shown in Fig. 2. The change in the flow rate of cold water is the input whereas the change in hot water outlet temperature is taken as the output. The input-output data is essential to acquire the model of the process. The experimentation is carried out for 1000s with a sampling time of 0.1s. The cold water flow rate is maintained between 0-0.12 lps.



**Fig 2: Response of the system for the PRBS input**

#### 5. SYSTEM IDENTIFICATION

System Identification is an approach for building mathematical models using the process input and output data for dynamic systems. System Identification identifies the models from input-output data and chooses the best model by comparing the performances of the model. Both the linear and nonlinear models are identified using System Identification techniques. In the linear case, both time-domain and frequency-domain data are supported whereas in the nonlinear case, only time-domain data is supported.

##### 5.1 Model Validation:

The identified models need to be validated, using some basic validation criteria. Here the satisfactory model is attained by validating the models from three different structures. The two structures include Box-Jenkins and Output- Error model which is derived from the Polynomial-linear models and the third structure is derived from Nonlinear ARX model structure. The accuracy of the models is based on three different validation criterias: Fitness in terms of %, FPE (Final Prediction Error) and Cost function.

##### 5.2 Input-Output Polynomial Models:

Polynomial models express the relationship between the output  $y(t)$ , the input  $u(t)$ , and the noise  $e(t)$  using equation (3). The variables A, B, C, D and F are polynomials specified in the time-shift operator  $q^{-1}$ . The variance of the white noise  $e(t)$  is  $\lambda$  (assumption) and  $n_a, n_b, n_c, n_d$  and  $n_f$  denotes the order of the polynomials.

$$A(q)y(t) = \frac{B(q)}{F(q)} u(t) + \frac{C(q)}{D(q)} e(t) \quad (3)$$

Where

$$A(q) = 1 + a_1q^{-1} + a_2q^{-2} + \dots + a_{na}q^{-na}$$

$$B(q) = b_1q^{-1} + b_2q^{-2} + \dots + a_{nb}q^{-nb}$$

$$C(q) = 1 + c_1q^{-1} + c_2q^{-2} + \dots + a_{nc}q^{-nc}$$

$$D(q) = 1 + d_1q^{-1} + d_2q^{-2} + \dots + a_{nd}q^{-nd}$$

$$F(q) = 1 + f_1q^{-1} + f_2q^{-2} + \dots + a_{nf}q^{-nf}$$

### 5.2.1 Output-Error Model:

The relation between input and undisturbed output  $w$  can be written as a linear difference equation. If the disturbances consists of white measurement noise, then we obtain the following description: [13]

$$w(t) + f_1w(t-1) + \dots + f_{nf}w(t-n_f) = b_1u(t-1) + \dots + b_{nb}u(t-n_b) \quad (4)$$

$$y(t) = w(t) + e(t) \quad (5)$$

$$\text{With } F(q) = 1 + f_1q^{-1} + \dots + f_{nf}q^{-nf} \quad (6)$$

The Output-Error model structure is given by

$$y(t) = \frac{B(q)}{F(q)} u(t) + e(t) \quad (7)$$

Output-Error polynomial model uses time or frequency domain data for estimation. This model is used when the dynamics are parameterized. In this case, the noise model is  $H=1$  and the white noise source  $e(t)$  affects only the output. The signal flow of this model is shown in Fig. 3.



Fig 3: Structure of Output-Error Model

### 5.2.2 Box-Jenkins Model:

A natural development of the output error model is to further model the properties of the output error. Describing this as an ARMA model gives

$$y(t) = \frac{B(q)}{F(q)} u(t) + \frac{C(q)}{D(q)} e(t) \quad (8)$$

The model set (Eq. 8) was suggested in Box and Jenkins (1970). This model also gives us the family of output-error related models. Box-Jenkins polynomial model uses time domain data for estimation. This model is entirely an independent parameterization for the dynamics and it provides added flexibility for modeling noise. The model structure for Box-Jenkins is shown in fig.4. [13]

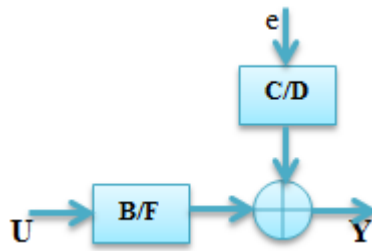


Fig 4: Structure of Box-Jenkins Model

5.3 Nonlinear ARX Model:

There are several nonlinearity estimators for nonlinear ARX models. Each nonlinearity estimator relates to an object class. Most nonlinearity estimators indicate the nonlinear function as a summer series of nonlinear units, such as wavelet or sigmoid networks. In this work, the Nonlinear ARX model uses the wavelet network for validation which has its class name “wavenet”. The identification methods for a nonlinear ARX model with parameters are nonlinear functions of input and output. The ARX model of the discrete-time linear system is represented by the equation

$$y[q] + a_1y[q - 1] + a_2y[q - 2] + \dots + a_ny[q - n] = b_1u[q - 1] + b_2u[q - 2] + \dots + b_nu[q - n] + c \quad (9)$$

Where  $y[q]$  and  $u[q]$  are output and input at time  $q$ ,  $a_i$  and  $b_i$  where  $i = 1, 2, \dots, n$  are coefficients, and  $n$  is the system order. The coefficients of nonlinear ARX model should depend on the values of output  $y[j]$ :  $j = q - n, q - n + 1, \dots, q$  and input  $u[j]$ :  $j = q - n, q - n + 1, \dots, q$  at time  $q$ . The structure of Nonlinear ARX model is shown in fig.5.

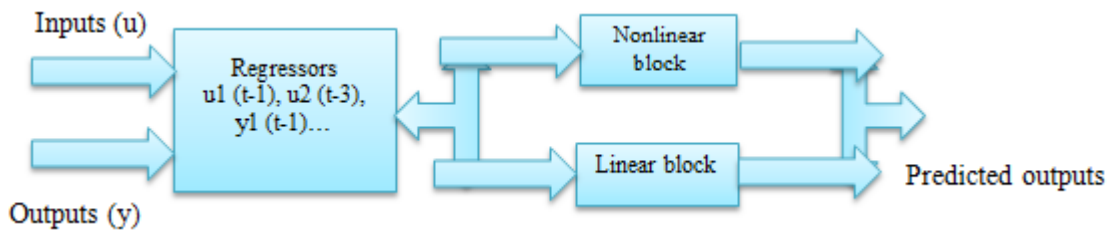


Fig 5: Structure of Nonlinear ARX Model

6. RESULTS AND DISCUSSIONS

The model attained from the response for PRBS signal is identified using three different identification techniques such as Nonlinear ARX, Box-Jenkins, and Output-Error model from which a comparatively best model is chosen by validating the model performance. By validating, the best model is chosen based on Percentage of Fitness, Final Prediction Error (FPE) and Cost Function. From fig.3 the best fitness is shown by Nonlinear ARX model which is said to have the highest fitness points of about 98.39%. Next to the Nonlinear ARX model, the best fit is given by Box-Jenkins with 88.33% and finally the Output-Error model with 77.96%. Fig.4 shows the Residual Analysis for three models. Autocorrelation and Cross-correlation of the residuals are shown here with 99% confidence intervals. Fig.5. shows the Nonlinear ARX model plot using wavenet class as the nonlinearity estimator. The performance criterias for the three models are tabulated in table 2.

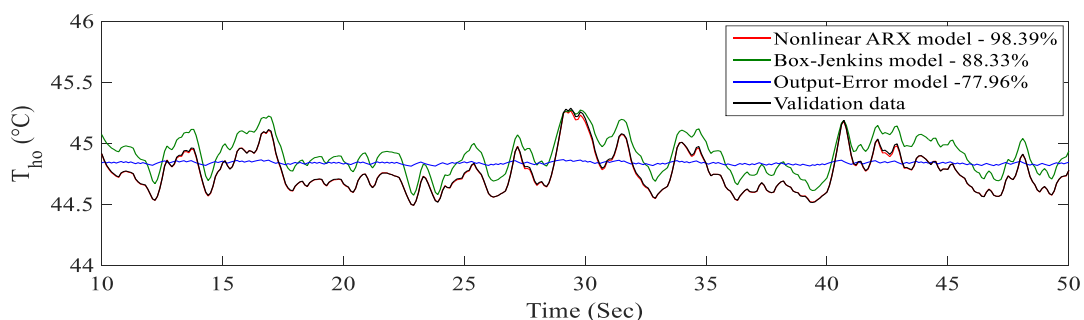
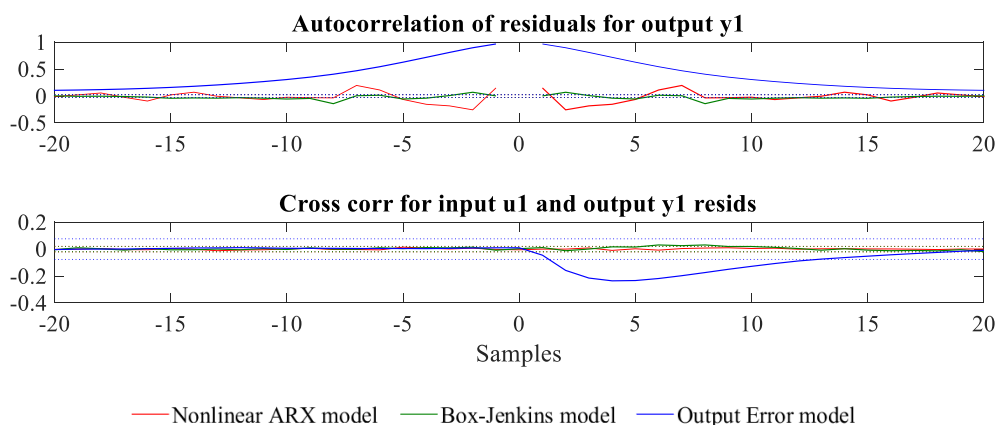
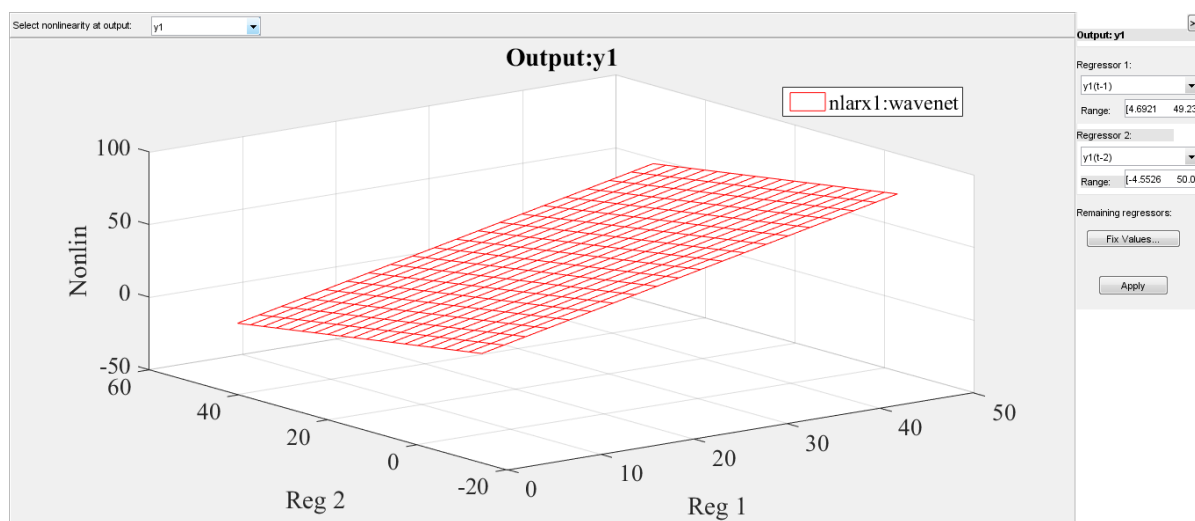


Fig 6: Sample of Measured and Simulated model output



**Fig 7: Residual Analysis of the three models**



**Fig 8: Nonlinear ARX model plot**

**Table 2: Performance Criterias**

Model	Fitness (%)	FPE	Cost Function
<b>Nonlinear ARX</b>	98.39	2.941e-05	2.69e-05
<b>Box-Jenkins</b>	88.33	6.15012e-05	6.14152e-05
<b>Output-Error</b>	77.96	0.03649	0.0364608

## 7. CONCLUSION

The PRBS Experiment is carried out to obtain the process input-output data. This data is essential in validating the performances using three models such as Nonlinear ARX, Box-Jenkins and Output-Error model. From the results shown, it is concluded that Nonlinear ARX is the best model than the other two models for validating the performance of the STHX system as it has the highest fitness points and the residual analysis for the same also covers the maximum of the data within the confidence intervals. Also the FPE and Cost Function for the nonlinear ARX model are small when compared with the other two model structures. The future scope includes designing advanced controllers for this temperature control process.

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